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# A Risk Measure for S-Shaped Assets and Prediction of Investment Performance

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## Abstract

In this paper, we study the option valuation of S-shaped assets. S-shaped assets are frequently encountered in technological developments, grant funding of research projects, and to a degree, hedge funds and stop-loss controlled trend-following investment vehicles. We conclude that the quantity  $\sigma^2/\mu$  (variance of return/expected return) replaced the traditional variance risk measure  $\sigma^2$  in the Black-Scholes option

valuation formula. We further study the interesting property of  $\sigma^2/\mu$  in predicting the turning point of performance of a portfolio of hedge funds in the early months of 2008 (and indeed, for earlier historical turning points).

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In technology development, it is well known that the S-curve descriptor is widely used in assessing the maturity of technology projects [Nieto et al. (1998)]. Our study is about the valuation of assets whose price changes follow the pattern of an S-shaped asset. That is, the asset prices either remain unchanged or go up at each time interval of observation.

In the equity investment world today, there are mainly two kinds of funds run by professionals:

- **Mutual funds** – these funds seek “relative return” [Harper (2003)], and their performance is measured against certain benchmarks, such as S&P 500, MSCI world index, or other indices. So if the index returned -5 percent and the fund returned -2 percent for the period, the fund is regarded as an excellent fund as it beats the market by 3 percent.
- **Hedge funds** – these funds seek “geared, absolute return,” and their performance is usually measured against 0 (or in some cases, against the risk-free bank deposit interest rate, which is much harder to achieve). Hence, at every report time window, the fund is expected to have a positive return (or in some cases, better than depositing the capital in the bank) compared to the end of the previous period. Consequently, in the case of hedge funds, negative returns are regarded as highly undesirable.

In real life, when we read the hedge fund monthly returns report, occasional monthly negative returns do appear. In the normal times, the negative returns are very rare, but during financial crises, we notice that the number of negative returns is more pronounced. Hence, we anticipate that if we model hedge funds using the concept of S-shaped assets during normal times, we may have interesting findings when we approach financial crises.

It has been widely noticed that the bond market feels financial crises before they happen. We are out to demonstrate, in this paper, using the three most recent financial crises as examples, that hedge funds feels financial crises before they take place with some clear measurable indicators.

In particular to this topic, we notice that in his report to the U.S. House of Representatives, Lo systematically analyzed the existing methods of studying systematic risks in the financial market, he postulated that the hedge funds should be regarded as a group, and suggested that researchers should use some systematic approaches to look at the risks that the financial markets are facing [Lo (2008)]. In particular, he quoted some of the network approaches in studying systematic risks in the hedge fund world. Taking the idea of systematic risks, we look at the hedge fund world from a different angle, but follow a similar approach: we use differently weighted portfolios of a large set of hedge funds and calculate the risk measure derived by us, to conclude that when approaching a financial crisis the hedge fund world as a whole begins to

deviate from the traditional “higher risk implies higher return” behavior. The “stress” in the pre-crisis financial market implies that “higher risks lead to lower returns.” This observation is not obvious when using a particular fund or index as observation reference, it can only be observed through a systematic approach as we explain below in detail.

In this paper, we follow the standard binomial formulation to establish the value of call/put options for S-shaped assets. Under appropriate assumptions, we further derive the approximate Black-Scholes (BS) European option pricing formula. It is interesting to note that in our situation, the involvement of the variance of the asset returns in the BS formulae is replaced by  $\sigma^2/\mu$  (variance of asset returns)/(expected return). And from this, we further reveal how hedge funds (as a group) anticipate the arrival of financial crises.

In our mathematical derivation, we use formal asymptotic analysis methods. Compared to standard BS formula derivation, the additional assumption for the variant Black-Scholes's approximation formula to hold is that  $\sigma^2/\mu$  is small compared to 1. Theoretically, this can be justified in S-shaped assets when the increase in the asset value is in one direction and is reasonably uniform. In this case, it is easy to demonstrate that  $\sigma^2 \propto \mu^2$ . Practically, this is justified by the hedge funds return data provided by International Asset Management (IAM) in non-financial-crisis periods (see graphs in the Appendix).

There have been many discussions in recent years as to why standard deviation is not an appropriate measure of risk. Newer risk measures such as Sharpe's Ratio, VaR [Linsmeier and Pearson (1996)], CVaR [Acerbi and Tasche (2002)], Omega [Shadwick and Keating (2002)], maximum drawdown [Chekhlov et al. (2003)] and many others have been introduced over the years. In particular, Tang and Yan (2010) observed that even using the standard binomial option value derivation formula, it is possible to derive “corrective” terms in addition to standard variance that indicate that the trend of asset price movement could get “tired” and turn in the opposite direction. These observations are backed up by real financial data testing. In this article, we argue that  $\sigma^2/\mu$  is a new kind of risk measure or a new starting point to search for a new class of risk functions. Some of the interesting behaviors it exhibits give it a new dimension in assessing absolute return assets.

We have discovered that for IAM selected quality hedge funds,<sup>2</sup> if we use Monte-Carlo methods to generate portfolio weights between 0 and 1 and plot the corresponding  $(\sigma^2/\mu, \mu)$  points, when the performance deteriorate, the shape of the cluster of points of  $(\sigma^2/\mu, \mu)$  begins to change shape

<sup>2</sup> Available selected funds are 16, 27, 38, 62, and 229 in periods Jan/96-Dec/98, Nov/99-Dec/01, Feb/01-Apr/04, Jun/03-Nov/05, and Jan/05-Jun/11 respectively.

dramatically with ample warning in advance – note that withdrawals from hedge funds usually face a three-month notice period or even temporary suspension in bad times, hence warnings in advance are particularly important. We point out that our prediction method is for hedge funds only and cannot be readily applied to other investment vehicles where absolute return is not the primary investment objective.

We also applied the same technique to draw similar conclusions for the 1998 and 2001 financial crises. Due to the fact that there were not as many hedge funds during those historical periods, we used all the hedge funds available at the time and we need to be aware that these are not the same data pool in each case.

### The assumptions concerning expected return and risk

We adopt the standard binomial formulation in our discussion. Assuming that:

1.  $\mu$  is the expected unit time period return
2. The volatility of the security is  $\sigma$
3.  $S_{\gamma}$  is the price of the security at  $t=\gamma$
4.  $S_{\Delta t}$  is assumed to be a random variable which either takes the value  $S_0 u$  ( $u > 1$  which is the up movement) with probability  $p$ , or stay at the value  $S_0$  with probability  $1-p$ .

Hence we have

$$E(S_{\Delta t}) = pS_0u + (1-p)S_0 \quad (1)$$

$$\text{Var}(S_{\Delta t}) = S_0^2 p(1-p)(u-1)^2 \quad (2)$$

Let  $\mu$  be the expected unit time period return,  $\sigma$  be the one time period risk of the asset. Using standard formulae that

$$E[(S_{\Delta t} - S_0)/S_0] \approx \mu \Delta t \Rightarrow E(S_{\Delta t}) \approx S_0(1 + \mu \Delta t)$$

$$\text{Var}[(S_{\Delta t} - S_0)/S_0] \approx \sigma^2 \Delta t \Rightarrow \text{Var}(S_{\Delta t}) \approx S_0^2 \sigma^2 \Delta t$$

We get

$$\{pS_0u + (1-p)S_0 \approx S_0(1 + \mu \Delta t), S_0^2 p(1-p)(u-1)^2 \approx S_0^2 \sigma^2 \Delta t \quad (3)$$

Asymptotically (when  $\Delta t$  is small), we can regard the above as equalities. Solving  $u$ , we obtain

$$u = 1 + \sigma^2/\mu + \mu \Delta t \quad (4)$$

We can also obtain the following relationships:

$$\begin{aligned} \mu \Delta t &= p(u-1) \\ \sigma^2 \Delta t &= p(1-p)(u-1)^2 \\ \sigma^2/\mu &= (1-p)(u-1) \end{aligned} \quad (5)$$

These relationships imply that there is at least one variable which can be set freely. We have to make the following assumptions to further our discussions:

**Assumption 1** – in our scenario where asset prices can only go up or stay stationary, we assume that the variance  $\sigma^2$  based on daily returns of the asset is a small quantity compared to  $\mu$ .

**Justification** –  $\mu$  is the expected unit time period return, it is usually a constant of a few percentage points. In our case where asset prices can only go up or stay stationary, the daily increase rate of the asset cannot exceed  $\mu$ . Since  $\sigma^2$  is the average of (daily return – average return)<sup>2</sup>, hence we can justify that, under usual circumstances where there is no high volatility in the underlying asset prices, we should expect:  $\sigma^2 \leq c\mu^2 \ll \mu$ , hence, when  $\Delta t$  is regarded as a small quantity

$$(u-1)^2 = (\sigma^2/\mu + \mu \Delta t)^2 \ll u-1 = \sigma^2/\mu + \mu \Delta t$$

This relationship cannot be directly implied from (5).

### Binomial formulae

#### Put option

In addition to the price movement patterns assumed above, we add the following assumptions: short selling is permitted; fractions of the security are permitted to be traded; there are no trading transaction costs and no dividend; and there are no arbitrage opportunities.

Denote by  $P_0$  the value of the put option on this security at  $t_0$ , and by  $P_+$  ( $P_-$ ) the corresponding option values at  $t = t_0 + \Delta t$  if the underlying prices goes up (stays the same):  $P_0 = P(S_0, t_0)$ ,  $P_+ = P(S_0u, t_0 + \Delta t)$ ,  $P_- = P(S_0, t_0 + \Delta t)$

It is clear that for put option,  $P_T = \text{Max}(E-S(T), 0)$ .

Now consider a portfolio consisting of one put option, and a short position of quantity  $\Xi$  to be specified later. We establish the value at  $t_0 + \Delta t$ : if the price has stayed, the portfolio has value  $P_- - \Xi S_0$ ; and if the price has moved up, the portfolio has value  $P_+ - \Xi S_0u$ .

We choose  $\Xi$  so that the portfolio has the same value in both cases:

$$P_- - \Xi S_0 = P_+ - \Xi S_0u, \Rightarrow \Xi = (P_- - P_+)/S_0(1-u) \quad (6)$$

We now have, using standard non-arbitrage theory,

**Proposition 1** – by the principle of non-arbitrage, we have

$$P_0 - \mathbb{E} S_0 = \text{Present Value Of } (P_- - \mathbb{E} S_0) \quad (7)$$

Substituting (6) into (7) and rearranging, we get

$$\begin{aligned} \{P_0 - \mathbb{E} S_0 + (P_- - \mathbb{E} S_0) \exp(-r\Delta t)\} &= (P_- - P_+)/\Delta t + [(P_+ - P_-)/\Delta t]e^{-r\Delta t}; \\ P_T &= \text{Max}[E-S(T), 0] \end{aligned} \quad (8)$$

where  $r$  is the risk-free interest rate.

The proof is to use standard arbitrage arguments and we omit the details [interested readers can refer to Higham (2004) and Willmot et al. (1995) for details].

### Call option

**Proposition 2** – similarly, let  $C$  be the value of the call option, using same notations, we have

$$\begin{aligned} \{C_0 - \mathbb{E} S_0 + (C_- - \mathbb{E} S_0) \exp(-r\Delta t)\} &= (C_- - C_+)/\Delta t + [(C_+ - C_-)/\Delta t]e^{-r\Delta t}; \\ C_T &= \text{Max}[S(T) - E, 0] \end{aligned} \quad (9)$$

where  $r$  is the risk-free interest rate.

### Asymptotic approximations for values of options

This section is rather mathematical. The main conclusion is that if we compare the differential equations derived for the standard Black-Scholes equation, we have  $\sigma^2/\mu$  in our equation replacing  $\sigma^2$  in the standard BS equation. This implies that  $\sigma^2/\mu$  is a more appropriate quantity in describing risks for S-shaped assets.

### Put option

Take put option as an example. Following the ideas developed in Friedman and Littman (1994), using (4) and (5), we have

$$\begin{aligned} P_+ &= P(S_0, t_0 + \Delta t) \\ &= P(S_0 + S_0(u-1), t_0 + \Delta t) \\ &= P(S_0, t_0) + \partial P / \partial S (S_0, t_0) S_0(u-1) + \frac{1}{2} \partial^2 P / \partial S^2 (S_0, t_0) S_0^2(u-1)^2 \\ &\quad + \partial P / \partial t (S_0, t_0) \Delta t + O(\Delta t^2) + O((u-1)^3) + O((u-1)\Delta t) \end{aligned}$$

$$\begin{aligned} P_- &= P(S_0, t_0) \\ &= P(S_0, t_0) + \partial P / \partial t (S_0, t_0) \Delta t + O(\Delta t^2) \end{aligned}$$

$$\begin{aligned} P_- - P_+ &= -\partial P / \partial S (S_0, t_0) S_0(u-1) - \frac{1}{2} \partial^2 P / \partial S^2 (S_0, t_0) S_0^2(u-1)^2 \\ &\quad + O(\Delta t^2) + O((u-1)^3) + O(\Delta t(u-1)) \end{aligned}$$

$$\begin{aligned} P(S_0, t_0) &= (P_- - P_+)/\Delta t + [(P_+ - P_-)/\Delta t]e^{-r\Delta t} \\ &= (P_- - P_+)/\Delta t + [(P_+ - P_-)/\Delta t]e^{-r\Delta t} + P_- e^{-r\Delta t} \end{aligned}$$

$$\begin{aligned} &= [(P_- - P_+)/\Delta t](1 - e^{-r\Delta t}) + P_- e^{-r\Delta t} \\ &= [\partial P / \partial S (S_0, t_0) S_0 + \frac{1}{2} \partial^2 P / \partial S^2 (S_0, t_0) S_0^2(1-u)]r\Delta t \\ &\quad + [P(S_0, t_0) + \partial P / \partial t (S_0, t_0) \Delta t](1-r\Delta t) + O(\Delta t^2) + O((u-1)^2\Delta t) \end{aligned}$$

Here we have used (5) in setting  $O(\Delta t^2/(u-1)) = O[(p/\mu) \Delta t] \leq O(\Delta t)$ .

Notice that from (5) and Assumption 1, the terms  $O(\Delta t^2)$  and  $O((u-1)^2\Delta t)$  are higher order terms compared to  $O(\Delta t)$ . Further simplifying the expression:

$$0 = [\partial P / \partial S (S_0, t_0) S_0 + \frac{1}{2} \partial^2 P / \partial S^2 (S_0, t_0) (1-u)]r\Delta t + \partial P / \partial t (S_0, t_0) \Delta t - P(S_0, t_0)r\Delta t + O(\Delta t^2) + O((u-1)^2\Delta t).$$

Dividing  $\Delta t$  on both sides, we get, using (4),

$$0 = [\partial P / \partial S (S_0, t_0) S_0 + \frac{1}{2} \partial^2 P / \partial S^2 (S_0, t_0) S_0^2(\sigma^2/\mu + \mu\Delta t)]r + \partial P / \partial t (S_0, t_0) - P(S_0, t_0)r + O(\Delta t) + O((u-1)^2).$$

Hence

$$0 = [\partial P / \partial S (S_0, t_0) S_0 + \frac{1}{2} \partial^2 P / \partial S^2 (S_0, t_0) S_0^2(\sigma^2/\mu)r + \partial P / \partial t (S_0, t_0) - P(S_0, t_0)r + O(\Delta t) + O((u-1)^2)].$$

Following Assumption 1, under normal circumstances, we can regard  $O((u-1)^2)$  as small quantities, hence we can drop the terms of order  $O(\Delta t)$  and  $O((u-1)^2)$  to get

$$\partial P / \partial S (S_0, t_0) S_0 r + \frac{1}{2} \partial^2 P / \partial S^2 (S_0, t_0) r S_0^2 \sigma^2/\mu - P(S_0, t_0) r + \partial P / \partial t (S_0, t_0) = 0$$

Using  $S_0, t_0$  as standard independent variables, we have shown

**Proposition 3** – under our assumptions, the put option value for our up-only asset is determined by

$$\begin{aligned} \partial P / \partial t + rS \partial P / \partial S + \frac{1}{2} r\sigma^2 S^2 / \mu \partial^2 P / \partial S^2 - Pr &= 0 \text{ with } P(T) = \\ \text{Max}[E-S(T), 0] \end{aligned} \quad (10)$$

This formula is also true when  $r$ , the risk-free interest rate, is dependent on  $t$  (possibly also on  $S$ ).

### Call option

Same assumptions and arguments lead to

**Proposal 4** – under our assumptions, the call option value for our up-only asset is determined by

$$\begin{aligned} \partial C / \partial t + rS \partial C / \partial S + \frac{1}{2} r\sigma^2 S^2 / \mu \partial^2 C / \partial S^2 - Cr &= 0 \text{ with } C(T) = \\ \text{Max}[S(T)-E, 0] \end{aligned} \quad (11)$$

This formula is also true when  $r$ , the risk-free interest rate, is dependent on  $t$  (possibly also on  $S$ ).

### The solutions to the PDE problems

Assume that  $r$ ,  $\sigma$ , and  $\mu$  are constants. We note that the expressions in (10) and (11) are similar to the Black-Scholes equation (replacing  $\sigma^2/\mu$  by  $\sigma^2$ , we will have Black-Scholes equations), and satisfy similar boundary conditions that:  $S \rightarrow 0 \Rightarrow P \rightarrow E$ ,  $C \rightarrow 0$ , and  $S \rightarrow \infty \Rightarrow P \rightarrow 0$ ,  $C \rightarrow S$ .

Following the same argument as for Black-Scholes equations, let  
 $d(S,t) = [2 \log(S/E) + (2 + \sigma^2/\mu)r(T-t)] \div 2\sigma\sqrt{r/\mu(T-t)}$

and

$$\Phi(z) = 1/\sqrt{2\pi} \int_{-\infty}^z \exp(-y^2/2) dy$$

for put option, we have

$$P(S,t) = E \exp[-r(T-t)] \Phi[\sigma\sqrt{r/\mu(T-t)} - d(S,t)] - S \Phi[-d(S,t)].$$

For call option, we have

$$C(S,t) = S \Phi[d(S,t)] - E \exp(-r(T-t)) \Phi[d(S,t) - \sigma\sqrt{r/\mu(T-t)}].$$

### How the S-risk function tells the trend change in a portfolio of hedge funds

In the previous sections, we have established an important fact: as far as option value is concerned, under not too volatile market conditions, our S-risk function  $\sigma^2/\mu$  is a more suitable measure of risk. Now we demonstrate how this new risk measure can be used to predict the performance trend change of a portfolio of hedge funds.

It is well known that one of the selling points of hedge funds is the absolute return. It is, therefore, possible to view hedge funds as a kind of S-shaped assets (in real life, this is not true but not too far from truth in a rational market – especially when market is calm and  $\sigma^2/\mu \ll 1$ ).

International Asset Management (IAM) is a fund of hedge funds based in London. IAM researches the hedge fund market and builds portfolios of hedge funds for its clients. IAM provided us with anonymized hedge fund returns data dated from November 1999 to June 2011, which covers the summer of 2008 when volatility of the financial market shot up.

We use the Monte-Carlo method to generate random nonnegative portfolio weights ( $w_1, w_2, \dots, w_{60}$ ) twenty thousand times such that  $w_1 + \dots + w_{60} = 1$ . Using one year historic data to calculate and plot the graph of  $(\sigma^2/\mu, \mu)$  (Figures 1 to 6).

These tell us that using our S-risk function  $\sigma^2/\mu$ , the warning signs would have been clear by January to February 2008. Taking into account the time it takes to arrive at a decision and an average of two months' notice

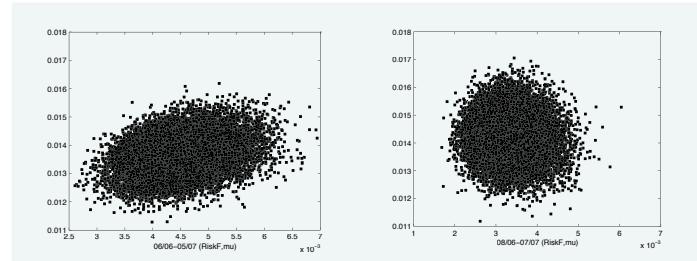


Figure 1 – Return against our risk function (the portfolios generated have performed reasonably. Risk-reward seems to be in proportion)

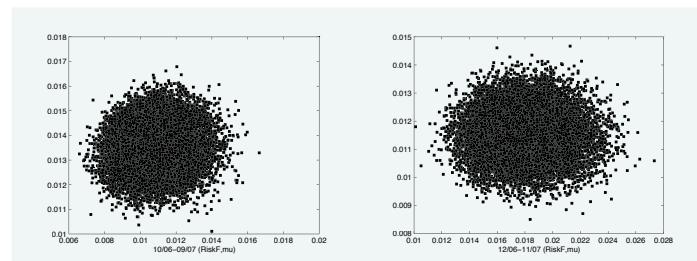


Figure 2 – Return against our risk function (The portfolios generated have performed reasonably. However, the second graph begins to show that larger risks may not bring higher returns)

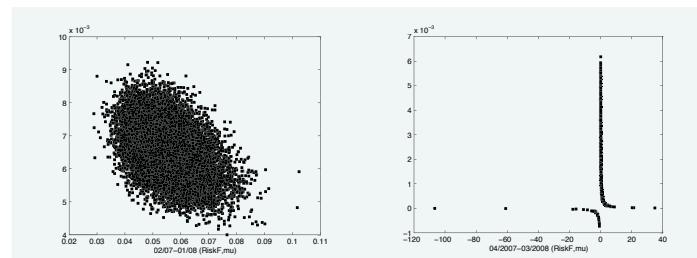


Figure 3 – Return against our risk function (The first graph shows the performance deterioration clearly: high risks seem to bring lower returns – bad sign for hedge funds. The second graph shows that the value of the risk measure becomes large and is no longer suitable to be used to measure risk)

period required for withdrawal, investors still would have been able to avoid the disaster for hedge funds performances of the second half of 2008 if they had taken action by March 2008.

Similar simulation has been applied to the 1998 and 2001 financial storms. During these periods, we have fewer hedge funds available, hence we have chosen all funds data that existed during the period (16 funds for 1997-1998, 27 funds available for 2000-2001) and did not make any selections using any criteria. The results show that (see Figures 4 to 6 at the end of the article):

1. For 1998, the warning signs became clear in June 1998 (using nine months back data, the warning signs appeared in April, but we need to investigate further to determine whether shorter historical data gives many more false warnings). The aftermath has seen some strong neg-

ative returns from the portfolios of the hedge funds. This again clearly indicated the Russian currency crisis and the beginning of the end of Long Term Capital Management. The recovery took place in May 1999 when risk and return became positively correlated again.

- For 2001, the warning signs became clear in November 2000. The impact of this downturn is much milder. In fact, the returns of the simulated portfolios remained in the positive territory, but the turning point shape of the  $(\sigma^2/\mu, \mu)$  graph has been persistent until May 2003. That means, for this considerable period of time, higher risk resulted in lower returns for the hedge fund portfolio. It is interesting that from November 2000 to May 2003, although higher risks resulted in lower returns, the actual returns of our Monte-Carlo portfolios never turned negative. Around June 2003 (shortly before and afterwards), our risk-return graph finally turned the trend to the “normal pattern” where higher risk implies statistical higher returns. The hedge fund industry really flourished around June 2003, many new funds were created and the “good times” had arrived.

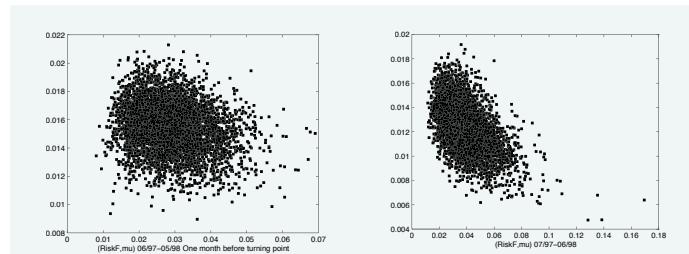


Figure 4 – The run up to the 1998 downturn

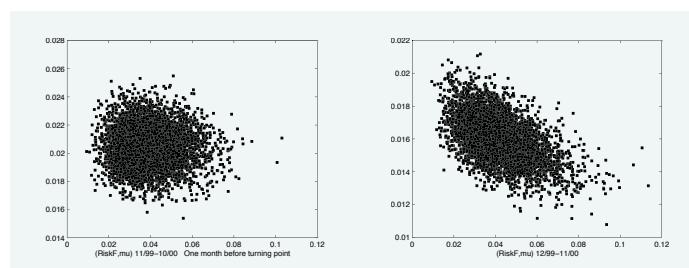


Figure 5 – The run up to the 2001 downturn

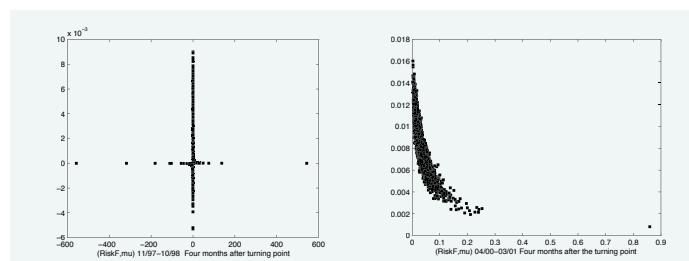


Figure 6 – The aftermaths (the 1998 (first graph) downturn turning point implied serious negative performance risk. The 2001 (second graph) downturn turning point implied more risks, but no significant negative performance risk as a portfolio)

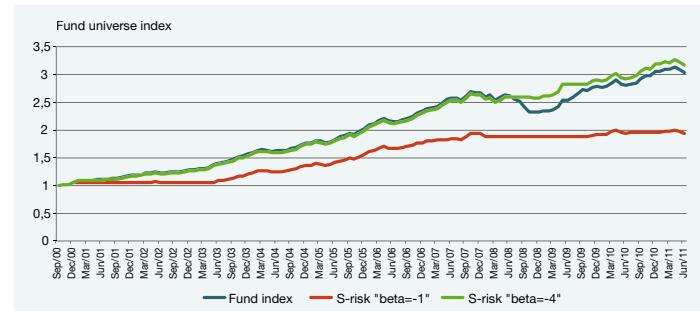


Figure 7 – The effect of S-risk function on hedge funds investment

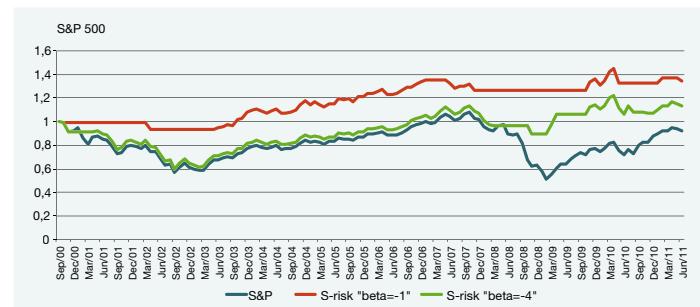


Figure 8 – The effect of S-risk function on S&P investment

Visual observations confirm the arrival of bad investment periods including the financial crisis in advance. Visual observations do not confirm patterns of distribution of  $(\sigma^2/\mu, \mu)$ , so we regressed  $\sigma^2/\mu$  on  $\mu$  of the samples generated by Monte Carlo. The coefficient  $\beta$  of  $\mu$  generated by the regression  $\sigma^2/\mu = \alpha + \beta\mu$  highlights the sensitivity of the S-risk function to the return. A negative  $\beta$  represents higher risk associated with lower return. A  $\beta$  value of -1 represents a unit return loss for one more unit of risk. We suggest that investors should shorten their investing periods and avoid some predicted bad periods when they receive the warning sign (i.e.,  $\beta < -1$ ) and wait until the sign recovers. The value of  $\beta$  depends on the investor's risk tolerance. We plot the effect of the S-risk function on investing one unit of money in the hedge funds universe index, which represents an equal weight ETF of the existing funds, and in S&P 500 for the period from September 2000 to June 2011. Note that for the S&P 500 investors can sell the stocks immediately while for hedge funds we allow two months' notice period to withdraw the money from the funds. There are no restrictions on buying.

From what we have discussed, it can be deduced that during the financial crisis of 2000 and 2008, hedge funds' performance as a whole falls far short of that of the stock market indices; however, it recovered very fast from the bottom. So the choice of -1 as a threshold leads to missing many good investing opportunities. We allow fewer restrictions on the sign and for comparison we choose a less strict threshold indicator of -4 to extend the investing time periods. We plot the effect of the S-risk

function using  $\beta$  of -1 and -4 for the hedge fund universe in Figure 7 and for S&P 500 in Figure 8. We can further rearrange the regression equation to get  $\sigma^2 = \alpha\mu + \beta\mu^2$ , which represents a reverse efficient frontier when  $\beta$  is negative as more risk leads to less return.

## Conclusion

The case where asset price cannot go down (or cannot go up) can often be used to approximate the cases of technology projects (the widely known S-curve theory) funding assessment and operational loss assessment. In these circumstances, the amount of investment that is required to support the project or to cure the cause of loss needs to be evaluated.

We propose this model as a first step in an effort to value these kinds of assets. The PDE model, with exact solution, is a good approximation of the binomial formulation. Due to the explicit solution formula, the PDE solution can be used in a much wider context much more efficiently. The most meaningful part of the discussion here is that we found that the risk measure  $\sigma^2/\mu$  is a suitable assessment for performances of assets with expected absolute returns. In this particular context, the quantity  $\sigma^2/\mu$  replaced the traditional  $\sigma^2$  in the Black-Scholes option value formula as an indicator of risk of the asset.

Our assumptions are simpler than that for the standard Black-Scholes equations. To make the simplification, we do have to add an empirical type condition that  $\sigma^2/\mu$  is small compared to 1. These conditions are usually satisfied when the market conditions are good. We give a theoretical justification/clarification (Assumption 1), which is further justified by the hedge funds data before the market turmoil – during the periods 06/06-05/07, 08/06-07/07, 10/06-09/07, and 12/06-11/07. It is clear that as market conditions deteriorated in 02/07-01/08 and 04/2007-03/2008, this assumption no longer holds. But correspondingly, the visual trend of funds performances also changes in time to give good warning about the market storm.

Finally we conclude that by using risk control, investments in hedge funds can be improved by 10 percent during 2007-2008 financial crisis. By contrast, investments in the S&P 500 can be improved by over 40 percent. This means that hedge funds as a whole, provide strong risk mitigating abilities when facing financial storms.

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